E 3322. *Proposed by Alexandru Lupaş, Sibiu, Romania.*

Suppose g is an even real-valued function on $[-a, a]$, suppose g is nondecreasing on $[0, a]$, and suppose h is a convex real-valued function on $[-a, a]$. Prove that

$$2a\int_{-a}^{a} g(t)h(t)\,dt \geqslant \int_{-a}^{a} g(t)\,dt\int_{-a}^{a} h(t)\,dt.$$

E 3323. *Proposed by Thane Plambeck, Stanford University, CA.*

A change-making machine gives change in chips of three integer denominations, 1 cent, α cents, and β cents, where $1 < \alpha < \beta$. The machine uses the following greedy algorithm: In making M cents change it dispenses chips of value β as long as possible, then chips of value α as long as possible, then chips of value 1 to complete the total of M. We say that a pair $\alpha < \beta$ is *frugal* if, for every M, the machine's algorithm dispenses the minimum number of chips among all ways of making change for M cents with denominations $1, \alpha, \beta$. (For example, $(2, 5)$ is frugal but $(4, 9)$ is not, because the $(4, 9)$ machine dispenses four chips in change for 12 cents instead of the minimum of three.)

For fixed $n > 2$, suppose the values $\alpha < \beta$ are two distinct integers chosen at random from $\{2, 3, \ldots, n\}$ with all pairs equally likely. Obtain an asymptotic formula (when n is large) for the probability that (α, β) is frugal.

E 3324. *Proposed by Mark D. Meyerson and R. Bruce Richter, U.S. Naval Academy, Annapolis, MD.*

Suppose that a and b are two closed arcs (homeomorphs of $[0, 1]$) in the plane, each of which contains the origin O but not as an endpoint. Prove or disprove: (a) there are arbitrarily small neighborhoods N of O such that $N \cap a$ is connected; (b) there are arbitrarily small neighborhoods N of O such that both $N \cap a$ and $N \cap b$ are connected.

SOLUTIONS OF ELEMENTARY PROBLEMS

Some Inequalities for Polynomials

E 2986* [1983, 133]. *Proposed by Abdul Aziz, University of Kashmir, India.*

If $P(z)$ is a polynomial of degree n with complex coefficients having all its zeros in $|z| \geqslant K$, where $K > 1$, prove or disprove the following two assertions:

(i) $$|P(K^2z) - P(z)| \leqslant (K^n - 1)\operatorname*{Max}_{|z|=1}|P(z)|, \text{ for } |z| = 1.$$

(ii) $$|P(K^2z)| - |P(z)| \leqslant (K^n - 1)|z|^n\operatorname*{Max}_{|z|=1}|P(z)|, \text{ for } |z| \geqslant 1.$$

Solution of (i) *by Paul Ilacqua, Santa Clara, CA.* Assertion (i) is not true in general. To see this take

$$P(z) = (z + K)^p(z - K),$$